

---

# Designing VHF Lumped-Element Couplers With MW Office

---

Steve Maas, Chief Technology Officer

## Applied Wave Research, Inc.

Copyright (C) 1999 Applied Wave Research, Inc.; All Rights Reserved.

---

### Abstract

---

This note describes two approaches to the design of broadband VHF lumped-element hybrid couplers. The first requires inductors having perfect coupling, while the second allows the use of imperfectly coupled inductors. MW Office is well suited to the design of both types of couplers.

### Introduction

---

The “classical” design for a lumped-element VHF quadrature hybrid was described 20 years ago by Ho and Furlow [1], [2]. This circuit, shown in Figure 1, uses a pair of perfectly coupled inductors (i.e., having a unity coupling coefficient) and two capacitors. The simple single-section coupler, however, is narrowband; connecting two of them by transmission lines increases the bandwidth substantially. Unfortunately, the transmission lines can be uncomfortably long, especially at low frequencies.

Design equations for this coupler are fairly simple. We first define the coupling ratio,  $k$ :

$$k = \left| \frac{S_{21}}{S_{41}} \right| \quad (1)$$

and the design equations are

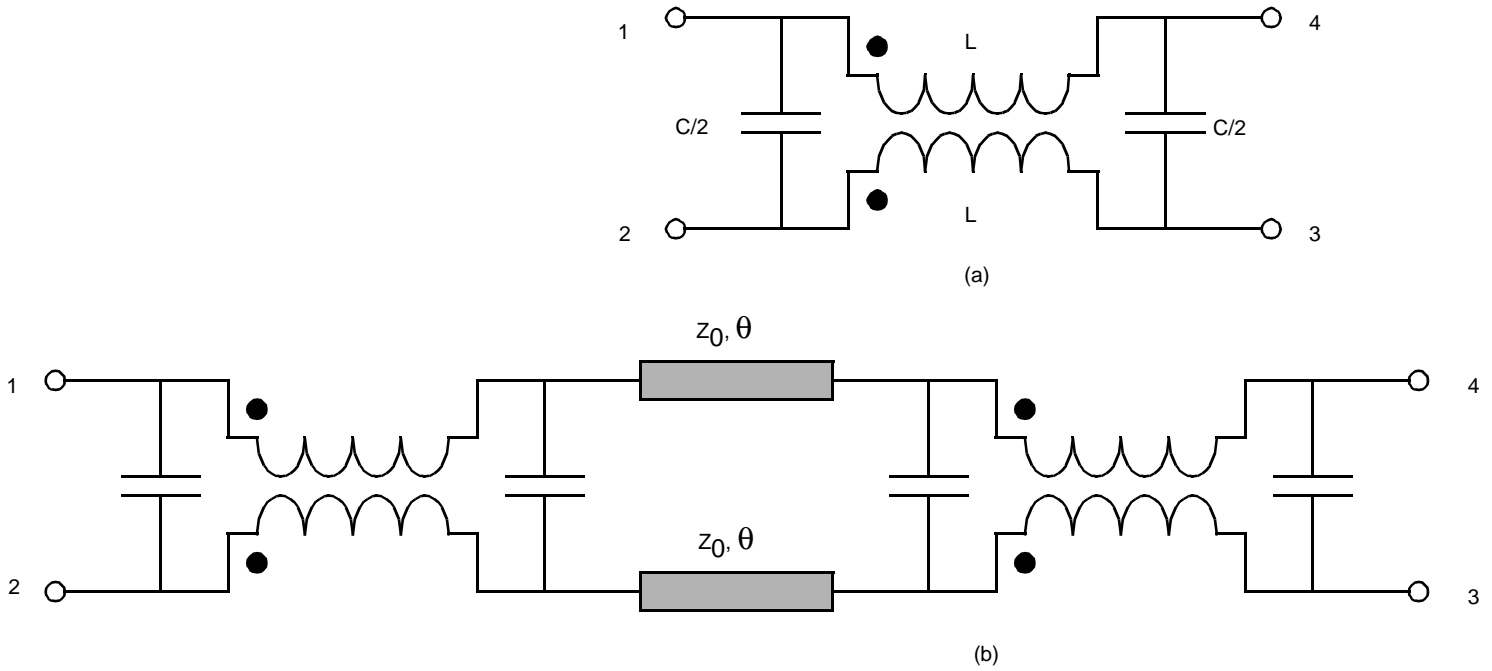


Figure 1. Single (a) and multisection (b) couplers from references [1] and [2]. The inductors are assumed to have perfect magnetic coupling and the transmission-line impedance  $Z_0$  is equal to the port impedance.

$$L = \frac{kZ_0}{\omega} \tag{2}$$

$$C = \frac{L}{Z_0^2}$$

where  $L$  and  $C$  are as shown in the figure, and  $\omega$  is the center frequency. Since the magnetic coupling must be perfect, the mutual inductance of the coupled inductors must equal the self inductance,  $L$ . In practice, tight coupling is achieved by winding the coils as a twisted pair of wires on a ferrite toroid.

A single-section coupler of this type is relatively narrowband. To increase the bandwidth, two identical sections can be connected by transmission lines, as shown in Figure 1(b). Ho and Furlow give a process for determining  $\theta$ , but it is a little cumbersome; a simpler method is to use MW Office's tuner to find  $\theta$ . To do this, simply enter  $\theta$  as a variable, make it tunable, and use the real-time tuner to optimize  $\theta$ .

The coupling is a little sensitive to the value of  $\theta$ ; slight changes in  $\theta$  result in an overcoupled or undercoupled response. Furthermore, the transmission

lines tend to shift the center of the coupling band lower in frequency. Thus, it is necessary to design the individual sections to have a center frequency somewhat higher than that of the finished coupler. MW Office's real-time tuner can be used to set the frequency precisely. Just enter the equations for  $L$  and  $C$  (Eq. (2)) and make  $\omega$  tunable. In a few seconds, you can adjust  $\theta$  and  $\omega$  for the optimum response.

When used as a 3-dB hybrid, the two-section coupler is capable of bandwidths of 60% or more. By overcoupling at the center frequency, the bandwidth can be increased somewhat further.

For a 3-dB hybrid coupler, typical transmission-line lengths are 25 - 45 degrees at the coupler's center frequency. At 350 MHz, for example, this is approximately 2 to 3 inches, depending on the dielectric constant of the substrate used for the transmission lines. In many applications, especially at lower frequencies, this is uncomfortably long. Clearly, it would be useful to have a broadband coupler design that did not require the transmission lines.

## An Alternate Design

---

It is possible to use an alternate design that requires no transmission lines, is broadband, and does not require perfect coupling between the inductors. As such, it may be useful in situations where toroidal inductors are impractical or where long transmission lines are intolerable. Figure 2 shows the new design.

The structure of the coupler approximates a set of coupled transmission lines. The self inductances of the inductors and the capacitors  $C_1$  approximate the individual lines, and the mutual inductance and capacitors  $C_2$  provide the coupling. Unfortunately, there must be both inductive and capacitive coupling; capacitive coupling alone is insufficient to obtain good performance. When the coupler is excited at port 1, the "through" port is port 4 and the coupled port is port 2. Port 3 is isolated.

Because its  $L$ - $C$  cutoff is too close to the center frequency of the coupler, the single-section structure, shown in Figure 2(a), is probably adequate only for narrowband applications. Usually a two-section coupler, Figure 2(b), is more satisfactory. The bandwidth can be extended slightly by using more sections, but the improvement is minor, probably not worth the additional cost.

The design equations are as follows. We first define the coupling  $K$  slightly differently from [1]:

$$K = |S_{21}| \quad (3)$$

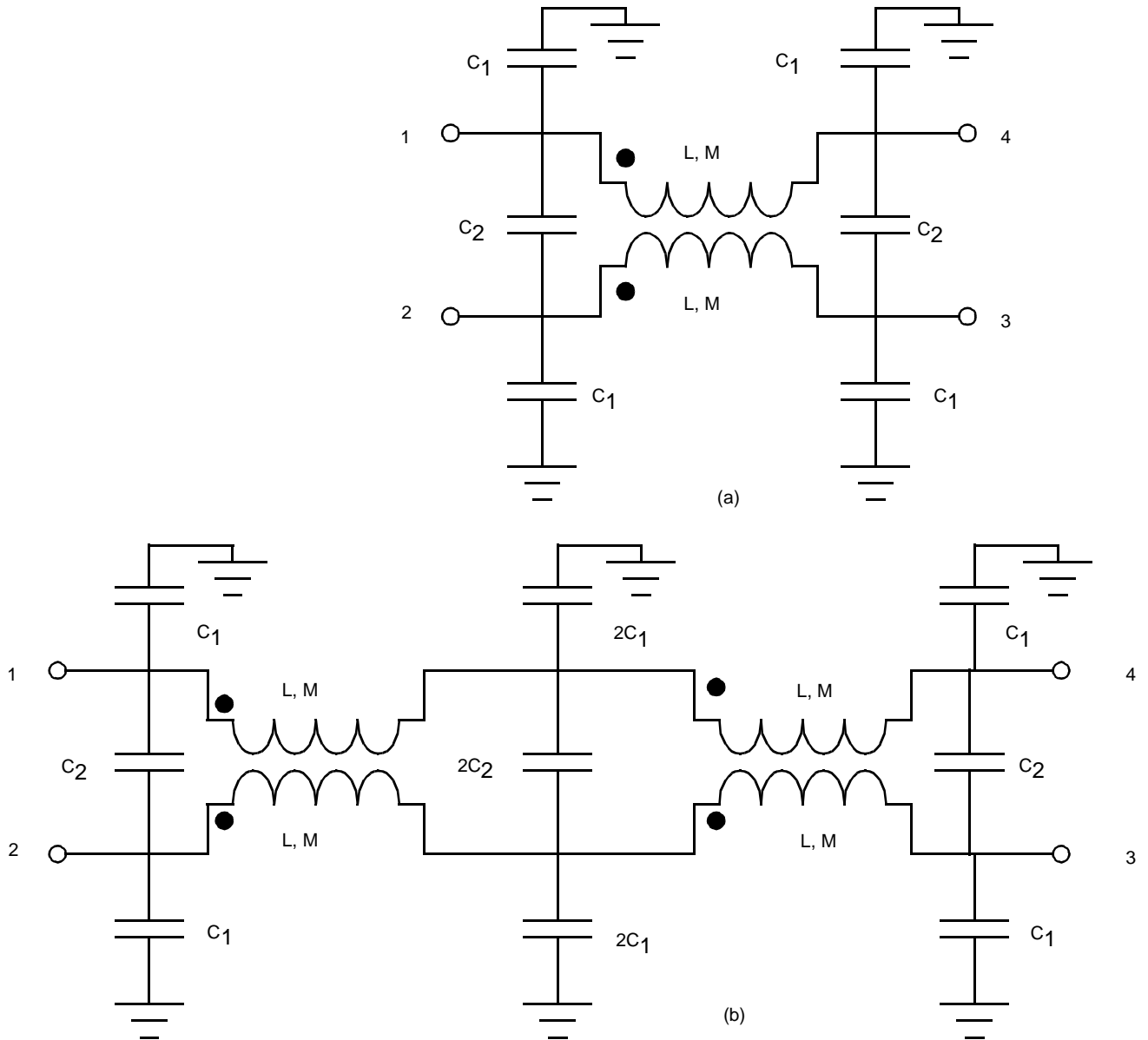


Figure 2. The circuit of the single-section coupler (a) and the two-section, (b). The inductors do not have perfect coupling. Because the L-C cutoff of the coupler is too close to the center frequency, the single-section coupler is probably not practical for most applications.

then we have

$$M = KL \tag{4}$$

$$C_1 = \frac{L - M}{2Z_0^2} \tag{5}$$

---

## Design Example

---

$$C_2 = \frac{MC_1}{(L-M)} = \frac{KC_1}{1-K} \quad (6)$$

where  $M$  is the mutual inductance, and  $C_1$ ,  $C_2$  and  $L$  are as shown in Figure 2. The center frequency  $f_0$  is

$$f_0 = \frac{1}{4n\sqrt{2(L+M)C_1}} \quad (7)$$

where  $n$  is the number of sections in the coupler. The approximate  $L$ - $C$  cut-off frequency of the coupler is

$$f_c = \frac{1}{\pi\sqrt{2(L+M)C_1}} \quad (8)$$

so the ratio of the cutoff frequency to the center frequency is simply

$$\frac{f_c}{f_0} = \frac{4n}{\pi} \quad (9)$$

Eq. (9) shows clearly why a single-section coupler ( $n = 1$ ) has poor characteristics; the cutoff frequency is only 27% above the center frequency! Because of this characteristic, a single-section 3-dB coupler has only about 25% bandwidth. However, a two-section coupler has  $f_c = (8 / \pi) f_0 = 2.54 f_0$ ; this is well outside the upper band edge of the coupler.

## Design Example

---

We design a two-section, 3-dB hybrid coupler centered at 355 MHz. First, from (3) we determine the coupling  $K = 0.707$ , and (4) gives the ratio of  $M$  to  $L$ . We select a trial value for  $L$ , then find  $M$  and  $C_1$  from (6). We substitute these values in (7) to determine the center frequency, then modify  $L$  and repeat the process until the center frequency is right. Finally, we calculate  $C_2$  from (6). After a couple of iterations, we obtain

$$L = 25 \text{ nH}$$

$$M = 17.7 \text{ nH}$$

$$C_1 = 1.47 \text{ pF}$$

$$C_2 = 3.52 \text{ pF}$$

Figure 3 shows the coupling, isolation, and return loss, calculated on the Microwave Office Design Suite [3]. The coupling decreases 0.5 dB at 245 and 435 MHz, giving a center frequency of 340 MHz and bandwidth of 56%. Figure 4 shows the phase. The phase balance between the “through” and coupled ports is well within one degree over this band. The return loss and isolation are nearly identical over this frequency range and are greater than 23 dB. Greater bandwidth could be achieved by slight overcoupling at the center frequency, reducing the coupling error at the band edges.

## Determining $L$ and $M$

---

It is difficult to design coupled inductors for specific values of  $L$  and  $M$ , but it is relatively easy to measure these parameters. Equations for the self inductances of the windings are well known, and those expressions can be used to design the individual windings. The distance between the windings can then be varied empirically to achieve the desired value of  $M$ .

The Z matrix of a pair of coupled inductors is

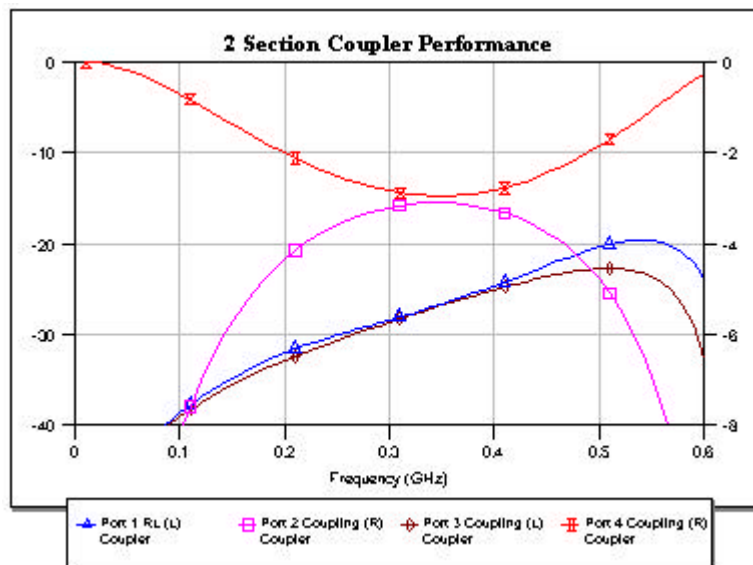


Figure 3. Coupling, return loss, and isolation of the 355-MHz, 3-dB coupler.

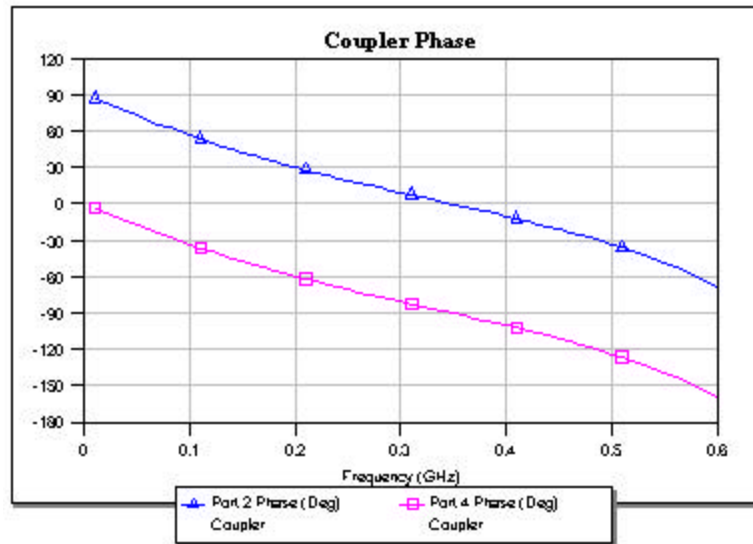


Figure 4. Phase of the 355-MHz coupler’s “through” and coupled ports.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega \begin{bmatrix} L & M \\ M & L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (10)$$

where the inductances of the two windings are identical, and the terminals of each winding realize a port. The pair of inductors is treated as a two-port and is measured with a network analyzer. The resulting S parameters are converted to Z parameters, and the self- and mutual inductances are found as

$$\begin{aligned} L &= \frac{Z_{11}}{j\omega} \\ M &= \frac{Z_{21}}{j\omega} \end{aligned} \quad (11)$$

It is best to calculate  $L$  and  $M$  from (11) over the entire frequency range of the coupler, and to average them to obtain values for use in the design. Large deviations from the average are an indication that the inductor may have too much stray capacitance.

This technique can be used in MW Office to determine  $L$  and  $M$  of planar spiral inductors. First, analyze the spiral inductor pair on the EM simulator. Then, simply plot the spiral’s  $Z_{11}$  and  $Z_{12}$  on a single graph, and on the same graph plot the Z parameters of a set of coupled inductors. Modify the values of  $L$  and  $M$  of the coupled inductors, using either the tuner or optimizer, until the Z parameters are the same. You will note that this can be done suc-

---

## References

---

cessfully only up to some maximum frequency; above this, resonances in the spiral cause the inductor to be nonideal. It is often possible to model these effects by adding additional elements, some of which may be “absorbed” into other elements of the coupler.

## References

---

- [1] C. Y. Ho and R. Furlow, “Design of VHF Quadrature Hybrids, Part I,” RF Design, July/August, 1979, p. 49.
- [2] C. Y. Ho and R. Furlow, “Design of VHF Quadrature Hybrids, Part II,” RF Design, September/October, 1979, p. 32.
- [3] Applied Wave Research, 1960 E. Grand Ave., Suite 530, El Segundo, CA 90278, USA.